Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 26 June 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
6669/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. Given that $y = \text{arsinh}(\tanh x)$, show that

$$\frac{dy}{dx} = \frac{\text{sech}^2 x}{\sqrt{1 + \tanh^2 x}}$$

(5)

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Question 1 continued

Q1

(Total 5 marks)

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2. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

The line l is the normal to E at the point $P(6\cos\theta, 5\sin\theta)$, where $0 < \theta < \frac{\pi}{2}$

(a) Use calculus to show that an equation of l is

$$6x\sin\theta - 5y\cos\theta = 11\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at the point Q.

The point R is the foot of the perpendicular from P to the x-axis.

(b) Show that $\frac{OQ}{OR} = e^2$, where e is the eccentricity of the ellipse E.

(4)

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Question 2 continued

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Question 2 continued

Q2

(Total 9 marks)

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3. (a) Using the definition for cosh x in terms of exponentials, show that

$$\cosh 2x \equiv 2\cosh^2 x - 1$$

(3)

(b) Find the exact values of x for which

$$29\cosh x - 3\cosh 2x = 38$$

giving your answers in terms of natural logarithms.

(6)

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Question 3 continued

Q3

(Total 9 marks)

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4. Use the substitution $x + 2 = u^2$, where $u > 0$, to show that

$$\int_{-1}^{7} \frac{(x+2)^{\frac{1}{2}}}{x+5}\,\mathrm{d}x = a + b\pi\sqrt{3}$$

where a and b are rational numbers to be found.

(9)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 9 marks)

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5. The plane Π_1 has equation $x - 2y - 3z = 5$ and the plane Π_2 has equation $6x + y - 4z = 7$

(a) Find, to the nearest degree, the acute angle between Π_1 and Π_2

(3)

The point P has coordinates $(2, 3, -1)$. The line l is perpendicular to Π_1 and passes through the point P. The line l intersects Π_2 at the point Q.

(b) Find the coordinates of Q.

(4)

The plane Π_3 passes through the point Q and is perpendicular to Π_1 and Π_2

(c) Find an equation of the plane Π_3 in the form $\mathbf{r} \cdot \mathbf{n} = p$

(4)

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Question 5 continued

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Question 5 continued

Q5

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6. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix}, k \in \mathbb{R}, k \neq \frac{1}{2}$$

(a) Show that det **M** = 1 – 2k.

(2)

(b) Find $\mathbf{M}^{-1}$ in terms of k.

(4)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix}$$

Given that l_2 has cartesian equation

$$\frac{x-1}{5} = \frac{y+2}{2} = \frac{z-3}{1}$$

(c) find a cartesian equation of the line l_1

(6)

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Question 6 continued

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Question 6 continued

Q6

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7.

$$I_n = \int_0^{\ln 2} \cosh^n x \, \mathrm{d}x, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \frac{3a^{n-1}}{nb^n} + \frac{n-1}{n} I_{n-2}$$

where a and b are integers to be found.

(6)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\ln 2} \cosh^4 x \, \mathrm{d}x$$

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 10 marks)

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8. The curve C has equation

$$y = \ln\left(\frac{e^x + 1}{e^x - 1}\right), \qquad \ln 2 \leqslant x \leqslant \ln 3$$

(a) Show that

$$\frac{dy}{dx} = -\frac{2e^x}{e^{2x} - 1}$$

(4)

(b) Find the length of the curve C, giving your answer in the form $\ln a$, where a is a rational number.

(6)

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Question 8 continued

Q8

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

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